# 第七届全国组合数论会议



主办单位: 河南大学数学与统计学院

2021年11月5日--8日

# 学术委员会(按姓氏排序)

陈永高(南京师范大学)

- 高维东 (南开大学)
- 洪绍方(四川大学)
- 孙智伟(南京大学)
- 袁平之(华南师范大学)

# 组织委员会(按姓氏排序)

陈士超 李亚莉 唐恒才 张伟 联系人: 陈士超 15937823940

# 腾讯会议

11月6日	会议	ID:	246	400	013
11月7日	会议	ID:	640	839	332
11月8日	会议	ID:	868	609	048

2021 年 11 月 6 日 腾讯会议 ID: 246 400 013				
时间	报告人	题目	主持人	
8:20-8:30	开幕式			
8:30-9:00	陈永高 南京师范大学	On a conjecture of Erdős and Lewin	孙智伟	
9:00-9:30	洪绍方 四川大学	On the sums of squares of exceptional units in residue class rings	南京大学	
9:30-9:45		茶歇		
9:45-10:10	千国有 四川大学	The distribution of divisors of polynomials	袁平之 华南师范大 学	
10:10-10:35	严慧芳 浙江师范大学	On the enumeration of simultaneous core partitions with restrictions		
10:35-10:50		茶歇		
10:50-11:15	侯庆虎 天津大学	Gosper summability of rational multiples of hypergeometric terms	王毅 大连理工大 学	
11:15-11:40	祝宝宣 江苏师范大学	Total positivity from the exponential Riordan arrays		
11:40-14:30 午间休息				
14:30-15:00	高维东 天津大学	Sums of sets of Abelian group elements	王天泽	
15:00-15:30	孙智伟 南京大学	On arithmetic properties of permanents	华北水利水 电大学	
15:30-15:45		茶 歇		
15:45-16:10	王六权 武汉大学	Sign changes of coefficients of powers of the infinite Borwein product	陈绍示 中科院数学 与系统科学 研究院系统 所	
16:10-16:35	陈 曦 大连理工大学	Analytic properties of combinatorial triangles related to Motzkin numbers		
16:35-16:45		茶歇	-	
16:45-17:10	杨全会 南京信息工程大学	On the values of representation functions		
17:10-17:35	邱敏 西华大学	The 3-adic valuations of Stirling numbers of the first kind #		
17:35-18:00	伍海亮 南京邮电大学	Elliptic curves and some cyclotomic matrices		

会议日程

2021 年 11 月 7 日 腾讯会议 ID: 640839332				
时间	报告人	题目	主持人	
8:30-9:00	袁平之 华南师范大学	Solutions to some classes of integer matrix equations	陈永高 南京师范大 学	
9:00-9:30	孙智宏 淮阴师范学院	Congruences for two types of Apéry-like sequences		
9:30-9:45	茶歇			
9:45-10:10	沙 敏 华南师范大学	Equational graphs over finite fields	孙智宏	
10:10-10:35	刘纪彩 温州大学	Some supercongruences on <i>q</i> -lacunary harmonic sums and <i>q</i> -trinomial	准 阴 师 泡 字 院	
10:35-10:50		茶歇		
10:50-11:15	陈绍示 中科院数学与系统 科学研究院系统所	Holonomic polynomial sequences I: degree growth	高维东	
11:15-11:40	韩冬春 西南交通大学	A tight upper bound for the maximal length of MDS elliptic codes	天津大学	
11:40-14:30 午间休息				
14:30-14:55	林志聪 山东大学(青岛)	Positivity and divisibility of enumerators of alternating descents	赵立璐 山东大学	
14:55-15:20	马无瑕 南京师范大学	Hegyvári's theorem on complete sequences		
15:20-15:35		茶歇		
15:35-16:00	郭军伟 淮阴师范学院	On a generalization of a congruence related to <i>q</i> -Narayana numbers 侯庆虏		
16:00-16:25	王晨 南京林业大学	Supercongruences concerning truncated hypergeometric series	天津大学	
16:25-16:40		茶歇		
16:40-17:05	范玉双 中国地质大学(北京)	Products of <i>k</i> atoms in Krull monoids		
17:05-17:30	秦小二 长江师范学院	二Constructing permutation polynomials via piecewise method千匹学院piecewise method四月强Integer sets with identical representation functions日		
17:30-17:55	陈世强 南京师范大学			

2021 年 11 月 8 日上午 腾讯会议 ID: 868609048					
时间	报告人	题 目	主持人		
8:30-8:55	何子龙 南方科技大学	On 2-universal integral quadratic forms over local fields	潘灏		
8:55-9:20	尼贺霞 南京审计大学	Proof of some supercongruences through a <i>q</i> -microscope	南京 <u>州</u> 纪八   学		
9:20-9:35		茶歇			
9:35-10:00	张汉斌 中山大学	Polynomial invariant theory of finite groups	张之正 · 洛阳师范学 院		
10:00-10:15	赵彤远 中国石油大学	On the <i>r</i> -positivity of multiset Eulerian polynomials			
10:15-11:00		闭幕式			

#### 报告摘要

#### Holonomic polynomial sequences I: degree growth

陈绍示 (中国科学院数学与系统科学研究院)

A sequence  $P_n(x)$  of polynomials in x is holonomic (P-recursive) if it satisfies a linear recurrence with polynomial coefficients in x and n. Many polynomial sequences from combinatorics, representation theory and number theory are shown to be holonomic. It is natural and fundamental to study the degree pattern of holonomic polynomial sequences. We will present a classification of the degree growth of such sequences and explain two applications related to combinatorial identities and exponential sums over finite fields respectively. This is a joint work with Jason P. Bell, Daqing Wan, Rong-Hua Wang and Hang Yin.

Integer sets with identical representation functions

陈世强(南京师范大学)

Let  $\mathbb{N}$  be the set of all nonnegative integers. For  $S \subseteq \mathbb{N}$  and  $n \in \mathbb{N}$ , let  $R_S(n)$ denote the number of solutions of the equation n = s + s',  $s, s' \in S$ , s < s'. In this talk, we prove that if m and r are two integers with  $m \ge 2$  and  $r \ge 0$ , A and B are two sets with  $A \cup B = \mathbb{N}$  and  $A \cap B = \{r + mk : k \in \mathbb{N}\}$ such that  $R_A(n) = R_B(n)$  for all positive integers n, then  $r = 2^{2l} - 1$  and  $m = 2^{2l+1} - 1$  for some positive integer l. This solves a problem posed by Chen and Lev in 2016. This is a joint work with professor Yong-Gao Chen.

Analytic properties of combinatorial triangles related to Motzkin numbers

#### 陈曦 (大连理工大学)

The Motzkin numbers count the number of lattice paths which go from (0,0) to (n,0) using steps (1,1), (1,0) and (1,-1) and never go below the x-axis.

Let  $M_{n,k}$  be the number of such paths with exactly k horizontal steps. In this talk, we investigate the analytic properties of various combinatorial triangles related to the Motzkin triangle  $[M_{n,k}]_{n,k\geq 0}$ , including their total positivity, the real-rootedness and interlacing property of the generating functions of their rows, and the asymptotic normality (by central and local limit theorems) of these triangles. We also prove several identities related to these triangles. This work is joint with Yi Wang and Sai-Nan Zheng.

On a conjecture of Erdős and Lewin

陈永高(南京师范大学)

A set A of positive integers is called d-complete if every sufficiently large integer is the sum of distinct terms taken from A such that no one divides the other. In this talk, we answer two questions of Erdős and Lewin partially and settle a conjecture of Erdős and Lewin on d-complete sequences affirmatively. We also pose two conjectures for further research. This is a joint work with Wang-Xing Yu.

Products of k atoms in Krull monoids

范玉双(中国地质大学(北京))

Let H be a Krull monoid with finite class group G such that every class contains a prime divisor. For  $k \in \mathbb{N}$ , let  $\mathscr{U}_k(H)$  denote the set of all  $m \in \mathbb{N}$ with the following property: There exist atoms  $u_1, \ldots, u_k, v_1, \ldots, v_m \in H$ such that  $u_1 \cdot \ldots \cdot u_k = v_1 \cdot \ldots \cdot v_m$ . It is well known that the sets  $\mathscr{U}_k(H)$ are finite intervals and  $\lambda_k(H) = \min \mathscr{U}_k(H)$  can be expressed in terms of  $\rho_k(H) = \max \mathscr{U}_k(H)$ . Moreover, the invariants  $\rho_k(H)$  depend only on G. If  $|G| \leq 2$ , then  $\rho_k(H) = k$  for every  $k \in \mathbb{N}$ . Suppose that  $|G| \geq 3$ . An elementary counting argument shows that  $\rho_{2k}(H) = kD(G)$  and  $kD(G) \leq \rho_{2k+1}(H) \leq kD(G) + \lfloor \frac{D(G)}{2} \rfloor$ , where D(G) is the Davenport constant. It is known that for cyclic groups we have  $kD(G)+1 = \rho_{2k+1}(H)$  for every  $k \in \mathbb{N}$ . We show that (under a reasonable condition on D(G)) for every noncyclic group there exists a  $k^* \in \mathbb{N}$  such that  $\rho_{2k+1}(H) = kD(G) + \lfloor \frac{D(G)}{2} \rfloor$  for every  $k \ge k^*$ . This is based on joint work with Qinghai Zhong.

Sums of sets of abelian group elements

For a positive integer k, let f(k) denote the largest integer t such that for every finite abelian group G and every zero-sum free subset S of G, if |S| = k then  $|\sum(S)| \ge t$ . We prove that  $f(k) \ge \frac{1}{6}k^2$ , which significantly improves a well known result of J.E.Olson. We also supply some other interesting results on f(k).

On a generalization of a congruence related to q-Narayana numbers

We study factors of some alternating sums of products of q-binomial coefficients related to q-Narayana numbers. Let  $\binom{n}{k}$  denote the q-binomial coefficients. We prove that for all positive integers  $n_1, \ldots, n_m, n_{m+1} = n_1$ , and j = 0 or 2m - 1, the alternating sum

$$\begin{bmatrix} n_1 + n_m + 1 \\ n_1 \end{bmatrix}^{-1} \sum_{k=-n_1}^{n_1} (-1)^k q^{jk^2 + \binom{k}{2}} \prod_{i=1}^m \begin{bmatrix} n_i + n_{i+1} + 1 \\ n_i + k \end{bmatrix} \begin{bmatrix} n_i + n_{i+1} + 1 \\ n_i + k + 1 \end{bmatrix}$$

is a polynomial in q with integer coefficients, and it has non-negative coefficients if m is odd. This partially confirms a conjecture of Guo and Jiang.

A tight upper bound for the maximal length of MDS elliptic codes

Determining the maximal length of MDS codes with certain dimension has been an interesting research topic in coding theory. The objective of this talk is to derive an upper bound for the maximal length of MDS elliptic codes over  $\mathbb{F}_q$  with dimension  $3 \leq k \leq \frac{q+1-2\sqrt{q}}{10}$ . For such a range of dimension k, our result improves an earlier bound of Munuera and gives an affirmative solution to the conjecture of Li, Wan, and Zhang. Most notably, the proposed upper bound is tight for odd k in the sense that it can be achieved by some well-designed MDS elliptic codes.

On 2-universal integral quadratic forms over local fields

#### 何子龙 (南方科技大学)

Let F be an algebraic number field. Recently, Xu and Zhang give the necessary and sufficient conditions for a binary (resp.ternary) indefinite quadratic  $\mathcal{O}_F$ -lattice to be locally universal, but not globally. Motivated by their work, we give an analogy of Conway and Schneeberger's 15-theorem for 2-universal quadratic lattices over local fields and the equivalent conditions for a quaternary locally 2-universal quadratic  $\mathcal{O}_F$ -lattice not to be globally 2-universal. In this talk, we will review known results on universal quadratic forms and then present our results. Also, we will briefly introduce the representation theory of quadratic forms over dyadic local fields, developed by Beli, and the theory of spinor genera. This is a joint work with Hu Yong and Xu Fei.

On the sums of squares of exceptional units in residue class rings

#### 洪绍方(四川大学)

Let  $n \ge 1, e \ge 1, k \ge 2$  and c be integers. An integer u is called a unit in the ring  $\mathbb{Z}_n$  of residue classes modulo n if gcd(u, n) = 1. A unit u is called an exceptional unit in the ring  $\mathbb{Z}_n$  if gcd(1-u, n) = 1. We denote by  $\mathcal{N}_{k,c,e}(n)$ the number of solutions  $(x_1, ..., x_k)$  of the congruence  $x_1^e + ... + x_k^e \equiv c$ (mod n) with all  $x_i$  being exceptional units in the ring  $\mathbb{Z}_n$ . In 2017, Mollahajiaghaei presented a formula for the number of solutions  $(x_1, ..., x_k)$  of the congruence  $x_1^2 + ... + x_k^2 \equiv c \pmod{n}$  with all  $x_i$  being the units in the ring  $\mathbb{Z}_n$ . Meanwhile, Yang and Zhao gave an exact formula for  $\mathcal{N}_{k,c,1}(n)$ . In this paper, by using Hensel's lemma, exponential sums and quadratic Gauss sums, we derive an explicit formula for the number  $\mathcal{N}_{k,c,2}(n)$ . Our result extends Mollahajiaghaei's theorem and that of Yang and Zhao. This is a joint work with Yulu Feng.

Gosper summability of rational multiples of hypergeometric terms

#### 侯庆虎 (天津大学)

By the telescoping method, Sun has recently given some hypergeometric series whose sums are related to  $\pi$  recently. We investigate these series from the point of view of Gosper's algorithm. Given a hypergeometric term  $t_k$ , we consider the Gosper summability of  $r(k)t_k$  for r(k) being a rational function of k. We give an upper bound and a lower bound on the degree of the numerator of r(k) such that  $r(k)t_k$  is Gosper summable. We also show that the denominator of r(k) can be read from the Gosper representation of  $t_{k+1}/t_k$ . Based on these results, we give a systematic method to construct series whose sums can be derived from the known ones. We also illustrate the corresponding super-congruences and the q-analogue of the approach.

Positivity and divisibility of enumerators of alternating descents

林志聪(山东大学(青岛))

The alternating descent statistic on permutations was introduced by Chebikin as a variant of the descent statistic. In this talk, we present some positivity and divisibility results and conjectures of alternating descent polynomials on permutations.

# Some supercongruences on *q*-lacunary harmonic sums and *q*-trinomial coefficients

In this talk, we present a variation of the q-Wolstenholme theorem, which extends the q-analogue of Wolstenholme's theorem due to Shi and Pan. In addition, we obtain a q-analogue of Sun's congruence on sums of central binomial coefficients, and a supercongruence for the q-trinomial coefficients, which proves a conjecture due to Apagodu and the author.

Hegyvári's theorem on complete sequences

Let A be a sequence of nonnegative integers. A sequence A is said to be *complete* if every sufficiently large integer can be represented as the finite sum of distinct terms of A. For a sequence  $S = \{s_1, s_2, \ldots\}$  and a real number  $\alpha > 0$ , let  $S_{\alpha} = \{\lfloor \alpha s_1 \rfloor, \lfloor \alpha s_2 \rfloor, \ldots\}, U_S = \{\alpha : S_{\alpha} \text{ is complete}\}$  and let  $\mu(U_S)$  be the Lebesgue measure of  $U_S$ . In 2013, Chen and Fang improved a result of Hegyvári in 1995 by proving that for  $1 < \gamma < 2$ , if  $s_{n+1} < \gamma s_n (n \ge n_0)$  and  $U_S \ne \emptyset$ , then  $\mu(U_S) > 0$  and proved that  $U_S \ne \emptyset$  if  $1 < \gamma < \frac{7}{4}$ . Recently, Fang and Liu showed that  $U_S \ne \emptyset$  if  $1 < \gamma < 1.898 \cdots$ . It is known that for any  $\gamma > 2$ , there exists a sequence S with  $s_n < s_{n+1} < \gamma s_n (n \ge n_0)$  and  $U_S = \emptyset$ . In this paper, we prove that  $U_S \ne \emptyset$  if  $1 < \gamma < 2$ . This gives an affirmative answer to a problem posed by Chen and Fang.

Proof of some supercongruences through a q-microscope

尼贺霞(南京审计大学)

Guo and Zudilin develop an analytical method, called "creative microscoping", to prove several supercongruences by establishing their q-analogues. We apply this method to give a q-Dwork-type generalization of Rodriguez-Villegas' supercongruences, which was recently conjectured by Guo and Zudilin. On the other hand, employing the same method, we confirm some similar supercongruences conjectured by Sun. The distribution of divisors of polynomials

千国有(四川大学)

Let F(x) be an irreducible polynomial with integer coefficients and degree at least 2. For  $x \ge z \ge y \ge 2$ , denote by  $H_F(x, y, z)$  the number of integers  $n \le x$  such that F(n) has at least one divisor d with  $y < d \le z$ . In this talk, we will introduce our results on the estimate of  $H_F(x, y, z)$ .

Constructing permutation polynomials via piecewise method

秦小二(长江师范学院)

Constructing permutation polynomials is a hot topic in finite fields, and permutation polynomials have many applications in different areas. In this talk, by using AGW criterion and piecewise method, we construct several classes of permutation polynomials with index q + 1 over  $\mathbf{F}_{q^2}$ .

The 3-adic valuations of Stirling numbers of the first kind

邱敏(西华大学)

Let *n* and *k* be positive integers. The Stirling number of the first kind, denoted by s(n,k), counts the number of permutations of *n* elements with *k* disjoint cycles. Let  $a \in \{1,2\}$ . In this talk, we will give an explicit formula on the 3-adic valuation of  $s(a3^n, k)$  with  $1 \le k \le a3^n$ . This gives an evidence to a conjecture of Hong and Qiu proposed in 2020. This is a joint work with Dr. Yulu Feng and Prof. Shaofang Hong.

Equational graphs over finite fields

沙敏(华南师范大学)

Given an equation E(X, Y) = 0 with variables X and Y over a finite field  $F_q$  of odd characteristic, we define a digraph by choosing the elements in  $F_q$ 

as vertices and drawing an edge from x to y if and only if E(x, y) = 0. We call this graph as equational graph, which generalises the notion of functional graph. When  $E(X,Y) = (Y^2 - f(X))(\lambda * Y^2 - f(X))$  with f(X) a polynomial over  $F_q$  and  $\lambda$  a non-square element in  $F_q$ , we show that if f is a permutation polynomial over  $F_q$ , then every connected component of the graph has a Hamiltonian cycle. Moreover, these Hamiltonian cycles can be used to construct balancing binary sequences. By making computations, it appears that almost all these graphs are strongly connected, and there are many Hamiltonian cycles in such a graph if it is connected.

Congruences for two types of Apéry-like sequences

#### 孙智宏(淮阴师范学院)

In this talk, we present many results and conjectures on congruences concerning the polynomials  $G_n(x)$  and  $V_n(x)$  given by

$$G_{n}(x) = \sum_{k=0}^{n} \binom{n}{k} (-1)^{k} \binom{x}{k} \binom{-1-x}{k},$$
$$V_{n}(x) = \sum_{k=0}^{n} \binom{n}{k} \binom{n+k}{k} (-1)^{k} \binom{x}{k} \binom{-1-x}{k}.$$

#### On arithmetic properties of permanents

孙智伟 (南京大学)

The permanent of an  $n \times n$  matrix  $[a_{j,k}]_{1 \leq j,k \leq n}$  over a field is defined by

$$\operatorname{per}[a_{j,k}]_{1 \leq j,k \leq n} = \sum_{\tau \in S_n} \prod_{j=1}^n a_{j,\tau(j)}.$$

In this talk we introduce recent studies of arithmetic properties of some permanents. For any primitive *n*-th root  $\zeta$  of unity, we obtain closed formulas for the permanents

per 
$$\left[1-\zeta^{j}x_{k}\right]_{1\leqslant j,k\leqslant n}$$
 and per  $\left[\frac{1}{1-\zeta^{j-k}x}\right]_{1\leqslant j,k\leqslant n}$ 

Another typical result states that for any odd integer n > 1 we have

$$t_n := \frac{1}{\sqrt{n}} \operatorname{per}\left[ \tan \pi \frac{jk}{n} \right]_{1 \le j, k \le (n-1)/2} \in \mathbb{Z},$$

and that  $t_p \equiv (-1)^{(p+1)/2} \pmod{p}$  for any odd prime p. We also pose several conjectures for further research; for example, we conjecture that

$$\operatorname{per}[|j-k|]_{1 \leq j,k \leq p} \equiv -\frac{1}{2} \pmod{p}$$

for any odd prime p.

Supercongruences concerning truncated hypergeometric series

In this talk, we introduce our recent work on supercongruences concerning truncated hypergeometric series. In particular, we shall report some supercongruences arising from the classical Karlsson-Minton summation formula. These are joint works with Hao Pan and Wei Xia.

Sign changes of coefficients of powers of the infinite Borwein product

We denote by  $c_t^{(m)}(n)$  the coefficient of  $q^n$  in the series expansion of  $(q;q)_{\infty}^m(q^t;q^t)_{\infty}^{-m}$ , which is the *m*-th power of the infinite Borwein product. Let t and m be positive integers with  $m(t-1) \leq 24$ . We provide asymptotic formula for  $c_t^{(m)}(n)$ , and give characterizations of n for which  $c_t^{(m)}(n)$  is positive, negative or zero. We show that  $c_t^{(m)}(n)$  is ultimately periodic in sign and conjecture that this is still true for other positive integer values of t and m. Furthermore, we confirm this conjecture in the cases (t,m) = (2,m), (p,1), (p,3) for arbitrary positive integer m and prime p.

Elliptic curves and some cyclotomic matrices

伍海亮(南京邮电大学)

In this talk, we will introduce some recent work on elliptic curves and related cyclotomic matrices. We will reveal the relationship between the numbers of rational points on certain elliptic curves over finite filed, Gaussian hypergeometric functions over finite fields and some cyclotomic matrices involving characters of finite fields.

On the enumeration of simultaneous core partitions with restrictions

严慧芳(浙江师范大学)

For a positive integer t, a partition is said to be a *t*-core partition, or simply a *t*-core, if it contains no box whose hook length is a multiple of t. The theory of *t*-core partitions lies at the intersection of a surprising number of fields, including number theory, combinatorics, and representation theory. Simultaneous core partitions have been extensively exploited after Anderson's work on the enumeration of (s, t)-core partitions. In this talk, we will present some results concerning the enumeration of simultaneous core partitions with restrictions.

On the values of representation functions

杨全会(南京信息工程大学)

Let  $\mathbb{N}$  be the set of nonnegative integers. For a set  $A \subseteq \mathbb{N}$ , let  $R_2(A, n)$ denote the number of solutions to a + a' = n with  $a, a' \in A$ , a < a'. In this talk, we will report some recent results on the values of  $R_2(A, n)$  when  $R_2(A, n) = R_2(\mathbb{N} \setminus A, n)$  from a certain point on. This is a joint work with Xing-Wang Jiang and Csaba Sandor. Solutions to some classes of integer matrix equations

Let  $M_m(\mathbb{Z})$  be the set of  $m \times m$  matrix over  $\mathbb{Z}$  where  $m \in \mathbb{N}$ . In this talk, by using the result of Fermat's Last Theorem and some quadratic equations, we show that the following second-order matrix equation has only trivial solutions:

$$X^{n} + Y^{n} = \lambda^{n} I \quad (\lambda \in \mathbb{Z}, \lambda \neq 0, X, Y \in M_{2}(\mathbb{Z})),$$

where X has an eigenvalue that is a rational number and  $n \in \mathbb{N}, n \ge 3$ ; By using the result of primitive divisors, we show that the second-order matrix equation

$$X^n + Y^n = (\pm 1)^n I \quad (n \in \mathbb{N}, n \ge 3, X, Y \in M_2(\mathbb{Z}))$$

has nontrivial solutions if and only if n = 4 or gcd(n, 6) = 1 and all nontrivial solutions are given; By constructing integer matrix, we also show that two matrix equation has an infinite number of nontrivial solutions. We also completely solve the second-order matrix equation  $X^2 + Y^2 = \lambda I, \lambda \in \mathbb{Z}$ .

Polynomial invariant theory of finite groups

#### 张汉斌 (中山大学)

Invariant theory is primarily concerned with the study of group actions and their fixed points. The actions are usually on algebras of various sorts, the fixed points are subalgebras with certain properties. In this talk, we consider linear actions of finite groups on polynomial algebras over a field (mainly the non-modular case). First, we present some recent studies on "degree bounds", a problem which can be traced back to classical works of E. Noether in the 1920s. Then we introduce some recent connections between invariant theory and combinatorial number theory. This talk is based on joint works with Dongchun Han. On the *r*-positivity of multiset Eulerian polynomials

赵彤远(中国石油大学(北京))

Let  $A_n^{(p)}(t)$  (resp.  $B_n^{(p)}(t)$ ) be the descent polynomial on permutations of the multiset  $\{1^p, 2^p, \dots, n^p\}$  (resp.  $\{1^p, 2^p, \dots, n^p, n+1\}$ ). The  $\gamma$ -positivity of  $A_n^{(p)}(t)$  was known but to give a combinatorial interpretation for the corresponding  $\gamma$ -coefficients still remains open. We manage to find a combinatorial interpretation for the  $\gamma$ -coefficients of  $A_n^{(p)}(t)$  via the model of weakly increasing trees; and prove that  $B_n^{(p)}(t)$  has bi- $\gamma$ -positivity expansion  $B_n^{(p)}(t) = a_n(t) + tb_n(t)$ , where  $b_n(t) = (p-1)A_n^{(p)}(t)$ . The first result, whose proof makes use of Chen's general bijective algorithm for trees and a new decomposition of weakly increasing trees, answers a recent open problem posed by Lin–Ma–Ma–Zhou. The latter result, which we provide both a computational proof and a short combinatorial proof, extends a bi- $\gamma$ positivity result due to Ma–Ma–Yeh from p = 2 to general p.

Total positivity from the exponential Riordan arrays

祝宝宣(江苏师范大学)

Many combinatorial triangles can be looked as the exponential Riordan arrays. In this talk, we will introduce some results for total positivity of the exponential Riordan arrays and Hankel total positivity of the row-generating polynomials. Furthermore, we also will present many such examples. Most results can be found in "Total positivity from the exponential Riordan arrays", to appear in SIAM Journal on Discrete Mathematics.